

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. MID SEMESTER EXAMINATION, SEPTEMBER 2012

FIRST YEAR

MATHEMATICS (General)

Date : 13/09/2012

Time : 11 am – 12 noon

Paper : I

Full Marks : 25

Group-A

1. Answer **any one** question: 1x3
- a) Find the modulus and principal amplitude of $\frac{1 + \sin \alpha - i \cos \alpha}{1 + \sin \alpha + i \cos \alpha}$ ($0 < \alpha < \pi$). 3
- b) If $x = e^{i\theta}$ and $1 + \sqrt{1 - a^2} = na$, prove that $\frac{a}{2n}(1 + nx)\left(1 + \frac{n}{x}\right) = 1 + a \cos \theta$. 3
2. Answer **any one** question: 1x3
- a) If $x_r = \cos \theta_r + i \sin \theta_r$ for $r = 1, 2, 3$ and $x_1 + x_2 + x_3 = x_1 x_2 x_3$, show that $\cos(\theta_1 - \theta_2) + \cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1) = -1$. 3
- b) Find the product of all values of $(1 + i\sqrt{3})^{\frac{3}{4}}$. 3
3. Answer **any one** question: 1x2
- a) Show that $\cos hz = \cos iz$ and $i \sin hz = \sin iz$. 2
- b) Show that $\cos h^2 z - \sin h^2 z = 1$. 2

Group-B

4. Answer **any two** questions: 2x2
- a) If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, find $(A \times B) \setminus (B \times A)$. 2
- b) Show that the mapping $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(n) = n + 2$, $n \in \mathbb{N}$ is one-to-one but not onto. 2
- c) A mapping $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined as follows:
 $f(x) = 1$, if x is rational
 $= -1$, if x is irrational.
Find the range of f . What is $f(\sqrt{7})$? 2
5. Find the eigen values of the following matrix:
- $$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 3 & 2 & -2 \end{pmatrix}$$

OR

State Cayley-Hamilton theorem and use it to find the inverse of $\begin{pmatrix} 1 & 3 \\ 0 & 4 \end{pmatrix}$. 4

Group-C

6. Answer **any two** questions:

2x2

a) Prove that $\sqrt{2}$ is an irrational number.

2

b) Find the limit: $\lim_{x \rightarrow 0} (1+2x) \frac{x+3}{x}$.

2

c) Show that the function given by

$$f(x) = \begin{cases} x \sin \frac{1}{x} : x \neq 0 \\ 0 : x = 0 \end{cases}$$

is continuous at $x = 0$.

2

7. Answer **any one** question:

1x5

a) (i) Show that $f(x) = \sin x$ is continuous at any point in \mathbb{R} .

2

(ii) Given $f(x) = \sqrt{x}$, $x \geq 0$. Prove that f is continuous at any point $c > 0$.

3

b) Define strictly monotone increasing function. Prove that every strictly monotone increasing function has inverse.

1+4